ANALYSIS OF THE EFFICIENCY OF RECTIFYING PLATES IN CROSS MOTION OF PHASES

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The efficiency of rectifying plates in cross motion of phases is analyzed. The equilibrium of vapor escaping from an ideal plate with a mean content of liquid on the plate equal to the mean value of vapor composition at the inlet and outlet from the plate is assumed. Four versions of interaction between an ideal and an actual plate are suggested. The equality of the efficiencies in vapor and liquid phases in each version is proved. Some versions are compared to the Murphree and Hausen models. The ratios between the efficiencies of the plates are obtained for the considered versions.

The complex nature of mass-transfer processes occurring on rectifying plates and diversity of designs and dimensions of the apparatuses cause difficulties in their calculation.

Rectifying plates with cross motion of phases are the most widely used in practice (Fig. 1). A liquid of the composition x_n that gets on the n-th plate interacts with vapor, is depleted by an easily volatile component, and leaves the plate with a concentration x_{n-1}. The concentrations of the easily volatile component in the liquid at the inlet to and outlet from the plate can greatly differ from each other, thus substantially affecting the composition of the vapor after the plate. Vapor with an initial composition y_{n-1} is enriched by the easily volatile component, then after the initial portion of the plate it has a composition y_{in}, and after the final portion it is y_{fin}, that is smaller than y_{in}. The mean composition of the vapor after the n-th plate is y_n. Along the path of the liquid on the plate, which considerably exceeds the height of the liquid layer on the plate, we can assume the absence of transverse mixing of liquid. Then, for an ideal plate which has an infinitely large surface of mass transfer, the vapor escaping from the initial portion should be in equilibrium with the liquid reaching the plate, and the vapor escaping from the final portion, with the liquid leaving the plate.

Equating the composition of the entire amount of vapor leaving an ideal plate to that of the vapor after the initial portion of it leads to overestimated values of the composition of the vapor after an ideal plate, and equating the composition of the vapor after the final portion of the plate, to underestimated. In turn, overestimated values of the composition of the vapor after an ideal plate cause lower values of the efficiency of an actual plate, and underestimated ones — higher values of the efficiency, which in a number of cases can exceed unity. This indicates that an actual plate operates more effectively compared to ideal.

Widely used models of mass transfer on the plate suggested in [1, 2] assume equilibrium of the vapor after an ideal plate with the composition of the liquid leaving it, thus leading to overestimated values of the efficiency of the plate, sometimes by an order and higher. Due to this, the models [1, 2] are applicable only to plates with intense mixing of the liquid on the plate or in a straight motion of vapor and liquid phases. In a cross motion of the components use of these models can lead to considerable errors.

In the model of [3-6] the equilibrium between the vapor leaving the plate and the liquid at the center of it is considered. Due to the difficulty of measurement of this concentration of the liquid in practice, the authors suggested substituting it with a mean value between the initial and final concentrations.

For a cross section of phases it is expedient to take into account also the composition of the vapor after the initial portion of the plate. With uniform depletion of the liquid during its motion on the ideal plate we can assume
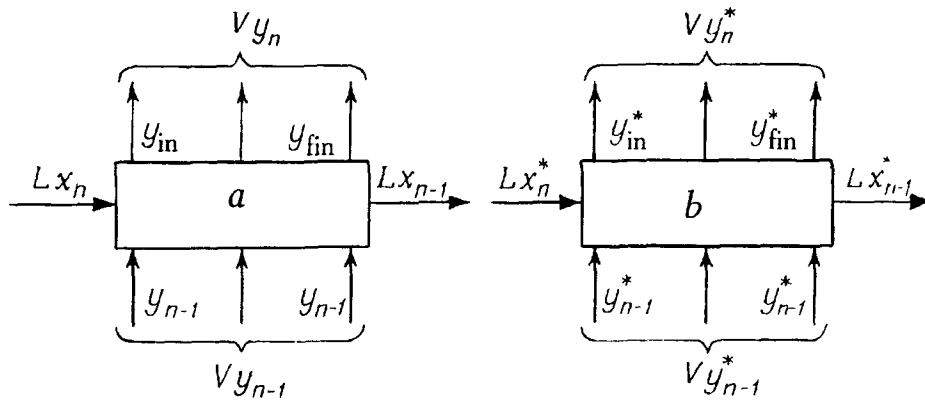

Fig. 1. Schematic of the cross motion of phases on actual (a) and ideal (b) plates.

a mean composition of the escaping vapor that is in equilibrium with the arithmetic-mean composition of the liquid on it, i.e.,

$$y_n^* = \frac{y_{in}^* + y_{fin}^*}{2} = \frac{m\ (x_{in}^* + x_{fin}^*)}{2}.$$

(1)

An analysis of possible relations between ideal and actual plates allows one to distinguish the versions that are shown in Fig. 2 and reflect the change in the concentrations of the easily volatile component in vapor and liquid phases during the motion of the liquid from the beginning to the end of the plate.

On the scheme of the change in the concentrations given in Fig. 2a, the incoming flows of the vapor and the escaping flows of the liquid have the same compositions for ideal and actual plates. This condition is typical of the Murphree model in an analysis of the efficiency in the vapor phase [1, 7]. In this scheme the equation of material balance for an ideal plate can be presented in the form

$$L\ [(x_n^*)_1 - x_{n-1}] = V\ [(y_n^*)_1 - y_{n-1}],$$

(2)

the efficiencies in the vapor and liquid phases being

$$E_{1\,mean,v} = \frac{y_n - y_{n-1}}{(y_n^*)_1 - y_{n-1}},$$

(3)

$$E_{1\,mean,liq} = \frac{x_n - x_{n-1}}{(x_n^*)_1 - x_{n-1}},$$

(4)

respectively, with relations (2) and (3) being similar to the Murphree model.

The relation between ideal and actual plates for the scheme presented in Fig. 2b is achieved under the condition of the equality of the composition of escaping flows of the vapor and the incoming flows of the liquid on both plates. In an analysis of the efficiency in the liquid [1, 7] both plates in the Murphree model have a similar relation. In contrast to the Murphree model, the equation of material balance for an ideal plate (Fig. 2b) can be presented as

$$L\ [x_n - (x_{n-1}^*)_2] = V\ [y_n - (y_{n-1}^*)_2].$$

(5)

The efficiency in the vapor phase is unlike, and in the liquid is the same as, that in the Murphree model
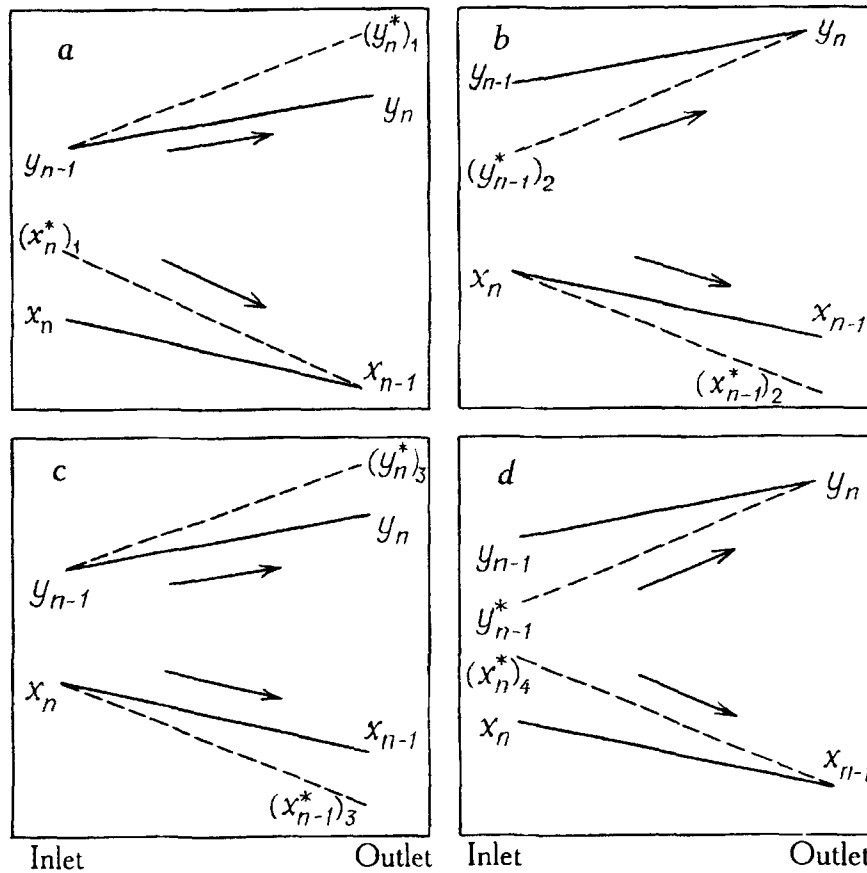
Fig. 2. Change in concentrations on actual (solid lines) and ideal (dashed lines) rectifying plates.

$$E_{2\text{mean},v} = \frac{y_n - y_{n-1}}{y_n - (y_{n-1}^*)_2}, \tag{6}$$

$$E_{2\text{mean,liq}} = \frac{x_n - x_{n-1}}{x_n - (x_{n-1}^*)_2}. \tag{7}$$

The condition of the relation between ideal and actual plates shown in Fig. 2c is the same as for the Hausen model — the flows of the vapor and the liquid reaching both plates are equal in composition and quantity [2, 7]. In this scheme, as in the Hausen model, the equation of material balance of an ideal plate

$$L [x_n - (x_{n-1}^*)_3] = V [(y_n^*)_3 - y_{n-1}] \tag{8}$$

and the expressions for the efficiency in vapor and liquid phases

$$E_{3\text{mean},v} = \frac{y_n - y_{n-1}}{(y_n^*)_3 - y_{n-1}}, \tag{9}$$

$$E_{3\text{mean,liq}} = \frac{x_n - x_{n-1}}{x_n - (x_{n-1}^*)_3} \tag{10}$$

also coincide.

For the scheme presented in Fig. 2d the equality of the compositions of the vapor and the liquid escaping from ideal and actual plates is typical. In this case the equation of material balance of an ideal plate has the form

$$L \left[(x_n^*)_4 - x_{n-1} \right] = V \left[y_n - (y_{n-1}^*)_4 \right],$$ (11)

the relations for the efficiency in the vapor phase and the liquid are

$$E_{4mean,v} = \frac{y_n - y_{n-1}}{y_n - (y_{n-1}^*)_4},$$ (12)

$$E_{4mean,liq} = \frac{x_n - x_{n-1}}{(x_n^*)_4 - x_{n-1}}.$$ (13)

Additional conditions of the relation between ideal and actual plates for the considered versions are the constancy of molar flows of the vapor and the liquid passing through both plates and the equality of heat transfer between the plates and the surrounding medium in both cases. For all versions the equation of material balance of an actual plate has the same form

$$L (x_n - x_{n-1}) = V (y_n - y_{n-1}).$$ (14)

In accordance with the conditions mentioned above, relation (1) for the considered versions (Fig. 2) acquires the form

$$(y_n^*)_1 = \frac{m \left[(x_n^*)_1 + x_{n-1} \right]}{2},$$ (15)

$$y_n = \frac{m \left[x_n + (x_{n-1}^*)_2 \right]}{2},$$ (16)

$$(y_n^*)_3 = \frac{m \left[x_n + (x_{n-1}^*)_3 \right]}{2},$$ (17)

$$y_n = \frac{m \left[(x_n^*)_4 + x_{n-1} \right]}{2}.$$ (18)

As a result of simultaneous solution of relations (2)-(4), (14), and (15) we can derive the relationship

$$x_n - x_{n-1} = \frac{E_{1mean,liq} \left(x_{n-1} - \dfrac{y_{n-1}}{m} \right)}{\dfrac{L}{mV} - \dfrac{1}{2}} = \frac{E_{1mean,v} \left(x_{n-1} - \dfrac{y_{n-1}}{m} \right)}{\dfrac{L}{mV} - \dfrac{1}{2}},$$ (19)

from which it follows that the efficiency in vapor and liquid phases for the version given in Fig. 2a are equal to each other, viz.,

$$E_{1mean} = E_{1mean,v} = E_{1mean,liq} = \frac{y_n - y_{n-1}}{(y_n^*)_1 - y_{n-1}} = \frac{x_n - x_{n-1}}{(x_n^*)_1 - x_{n-1}}.$$ (20)

Simultaneous solution of relations (5)-(7), (14) and (16) allows one to obtain the expression

$$x_n - x_{n-1} = \cfrac{x_{n-1} - \dfrac{y_{n-1}}{m}}{\dfrac{L}{mV} - 1 + \dfrac{1}{2E_{2mean,v}}} = \cfrac{x_{n-1} - \dfrac{y_{n-1}}{m}}{\dfrac{L}{mV} - 1 + \dfrac{1}{2E_{2mean,liq}}},$$ (21)

which confirms the equality of the efficiencies in vapor and liquid phases (Fig. 2b)

$$E_{2mean} = E_{2mean,v} = E_{2mean,liq} = \frac{y_n - y_{n-1}}{y_n - (y_{n-1}^*)_2} = \frac{x_n - x_{n-1}}{x_n - (x_{n-1}^*)_2}.$$ (22)

Solving expressions (8)-(10), (14), and (17) simultaneously, we can obtain the relationship

$$x_n - x_{n-1} = \cfrac{E_{3mean,v}\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)}{\dfrac{L}{mV} + \dfrac{1}{2} - E_{3mean,v}} = \cfrac{E_{3mean,liq}\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)}{\dfrac{L}{mV} + \dfrac{1}{2} - E_{3mean,liq}},$$ (23)

from which it also follows that the efficiencies in the vapor phase and the liquid for the version presented in Fig. 2c are equal:

$$E_{3mean} = E_{3mean,v} = E_{3mean,liq} = \frac{y_n - y_{n-1}}{(y_n^*)_3 - y_{n-1}} = \frac{x_n - x_{n-1}}{x_n - (x_{n-1}^*)_3}.$$ (24)

The solution of expressions (11)-(14) and (18) leads to the relation

$$x_n - x_{n-1} = \cfrac{x_{n-1} - \dfrac{y_{n-1}}{m}}{\dfrac{L}{mV} - \dfrac{1}{2E_{4mean,v}}} = \cfrac{x_{n-1} - \dfrac{y_{n-1}}{m}}{\dfrac{L}{mV} - \dfrac{1}{2E_{4mean,liq}}},$$ (25)

which, as previous similar ones, confirms the equality of the efficiencies in the vapor and the liquid for the scheme depicted in Fig. 2c:

$$E_{4mean} = E_{4mean,v} = E_{4mean,liq} = \frac{y_n - y_{n-1}}{y_n - (y_{n-1}^*)_4} = \frac{x_n - x_{n-1}}{(x_n^*)_4 - x_{n-1}}.$$ (26)

Thus, for each of the considered versions, the efficiencies in the vapor phase and the liquid are equal, what takes place in the Hausen model and distinguishes the first and the second versions from the Murphree model.

On the left of relations (19), (21), (23), and (25) the difference of the concentrations of the easily volatile component in the liquid reaching the actual plate and draining from it, i.e., the same technological result, is shown. Equating the right sides of these relations with account for (20), (22), (24), and (26) allows one to obtain relations between the efficiencies of all the considered versions of mass transfer in cross motion of the components

$$\frac{\dfrac{L}{mV} - \dfrac{1}{2}}{E_{1mean}} = \frac{L}{mV} + \frac{1}{2E_{2mean}} - 1 = \frac{\dfrac{L}{mV} + \dfrac{1}{2}}{E_{3mean}} - 1 = \frac{L}{mV} - \frac{1}{2E_{4mean}}.$$ (27)

Analysis of the efficiencies of the plates shows that they coincide for some versions. In particular, similar formulas have expressions for the efficiency in the vapor phase (3) and (9), (6) and (12) and in the liquid (4) and (13), (7) and (10). However, no equality of the values of the efficiencies determined by the above-given formulas follows from this. It is expedient to consider the mentioned groups of expressions in combination with the corresponding equations of material balances of ideal plates and also with the conditions of the equilibrium of
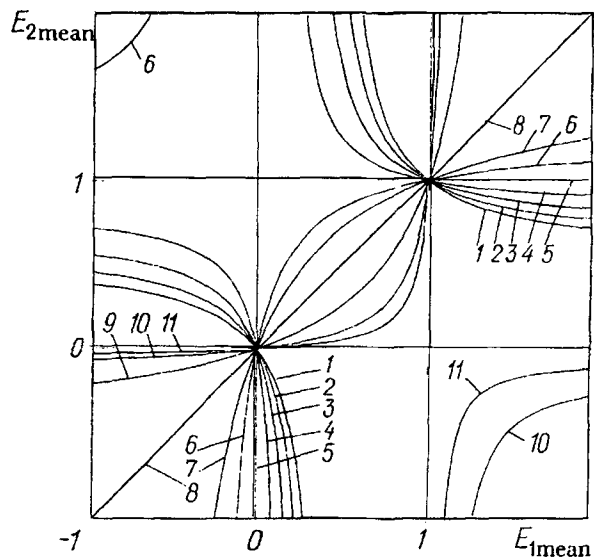
Fig. 3. Dependence of the efficiencies E_{1mean} and E_{2mean} at different values of L/mV: 1) 0.1, 2) 0.2, 3) 0.3, 4) 0.4, 5) 0.5, 6) 0.6, 7) 0.7, 8) 1.0, 9) 2.0, 10) 5.0, 11) 10.0.
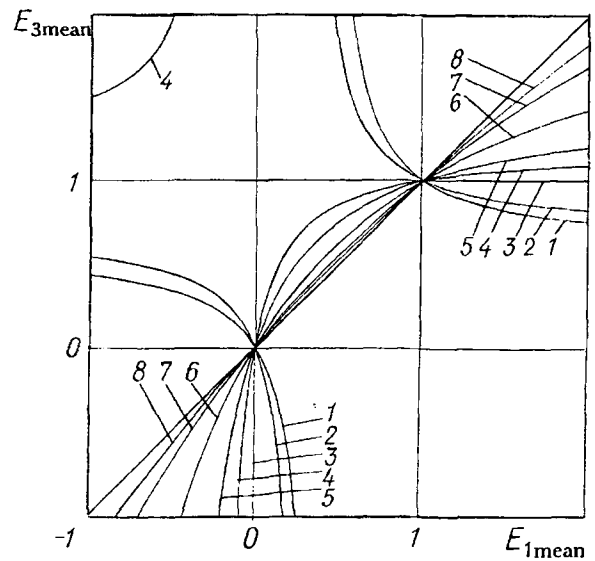
Fig. 4. Dependence of the efficiencies E_{1mean} and E_{3mean} at different values of L/mV: 1) 0.1, 2) 0.2, 3) 0.5, 4) 0.7, 5) 1.0, 6) 2.0, 7) 5.0, 8) 10.0.
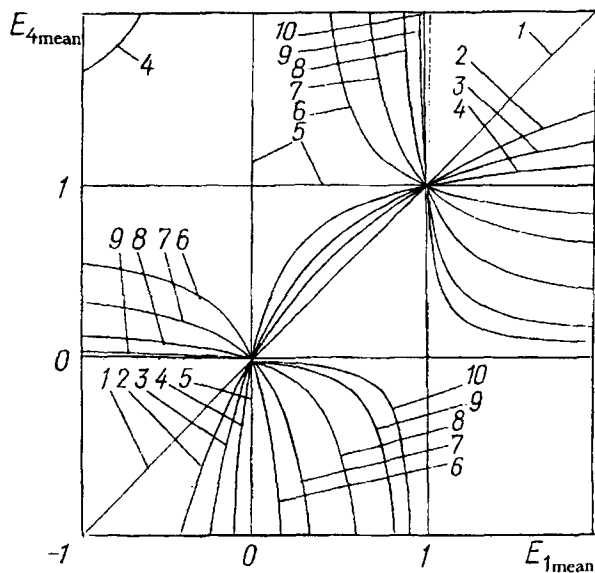

Fig. 5. Dependence of the efficiencies E_{1mean} and E_{4mean} at different values of L/mV: 1) 0.0, 2) 0.2, 3) 0.3, 4) 0.4, 5) 0.5, 6) 0.7, 7) 1.0, 8) 2.0, 9) 5.0, 10) 10.0.
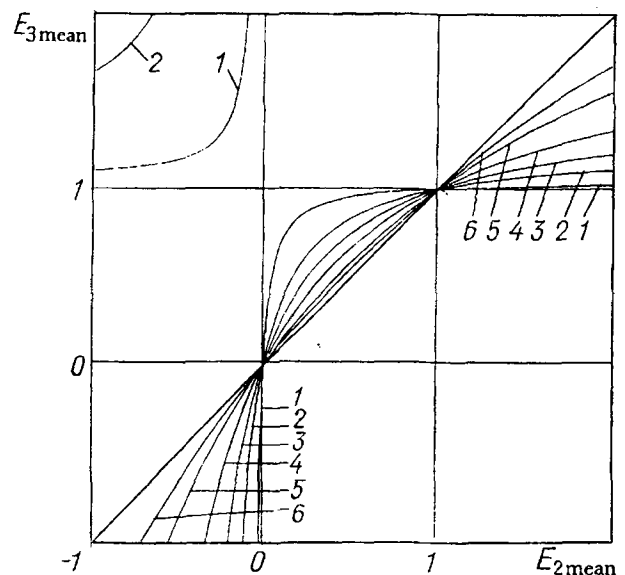
Fig. 6. Derpendence of the efficiencies E_{2mean} and E_{3mean} at different values of L/mV: 1) 0.1, 2) 0.5, 3) 1.0, 4) 2.0, 5) 5.0, 6) 10.0.

compositions which are individual for each version. In particular cases at certain ratios of flows and concentrations the coincidence of the values of the efficiency of some versions is possible.

Figures 3-8 present graphs of the interrelations between individual pairs of efficiencies at some values of L/mV.

As is seen from Fig. 3, the values of the efficiency E_{2mean} are higher than those of E_{1mean} within the range $0.5 < L/mV < 1$, lower for $L/mV > 1$, and coincide at $L/mV = 1$. At $L/mV < 0.5$ the relations of the efficiencies E_{1mean} and E_{2mean} are beyond real values of one of them. So, for example, efficiencies E_{2mean} smaller than zero or

741
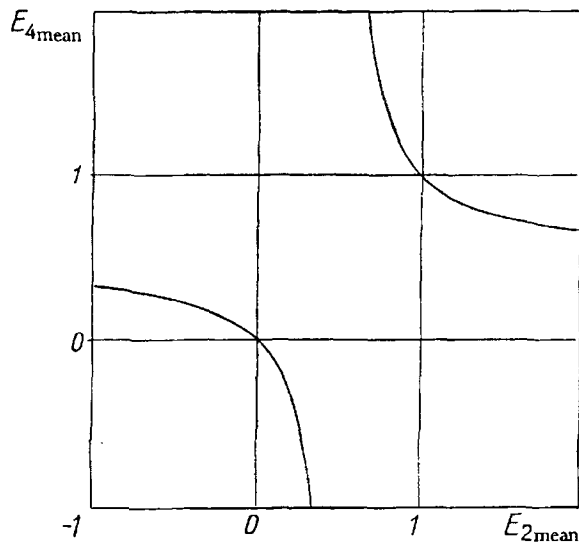
Fig. 7. Interrelation between the efficiencies E_{2mean} and E_{4mean}.
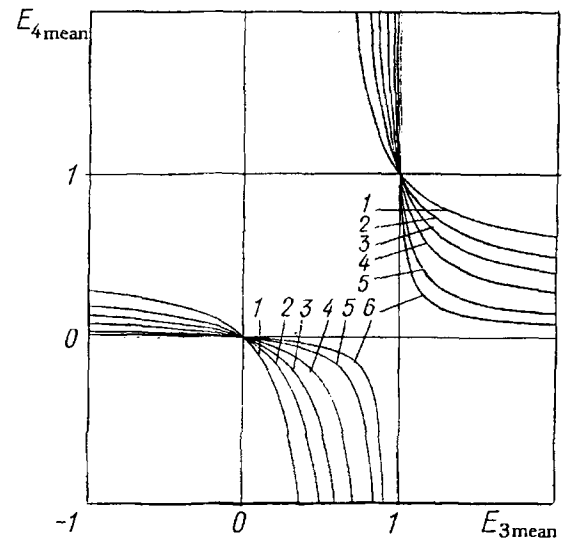
Fig. 8. Dependence of the efficiencies E_{3mean} and E_{4mean} at different values
of L/mV: 1) 0.1, 2) 0.5, 3) 1.0, 4) 2.0, 5) 5.0, 6) 10.0.

higher than unity correspond to the real range $0 < E_{1mean} < 1$, while the real efficiency in the range $0 < E_{2mean} < 1$ is observed at negative E_{1mean} or at $E_{1mean} > 1$. This fact can be useful in the interpretation of experimental studies when the efficiency of the plate on use of one of the two considered models of mass transfer acquires unreal values which defy common sense. In this case it is expedient to use another model.

A comparison of the efficiencies E_{1mean} and E_{2mean} shows that the first is smaller than the second for $L/mV > 0.5$ (Fig. 4). This difference becomes less as L/mV increases and virtually disappears in the region $L/mV > 2$. At $L/mV < 0.5$, as in the previous case, to the real range of one of the efficiencies there correspond values of the other that are lower than zero or higher than unity; this can also be used in obtaining unreal values of one of the efficiencies in one of the mentioned models of mass transfer.

In contrast to the two previous cases a relation between the real efficiencies E_{1mean} and E_{4mean} is possible only in the region of $L/mV < 0.5$ (Fig. 5). The difference between these efficiencies decreases with L/mV and the first of them is smaller than the other. At $L/mV > 0.5$ real values of one of the efficiencies occur when the other is unreal; this can also be used in the interpretation of experimental data.

Study of the efficiencies E_{2mean} and E_{3mean} (Fig. 6) shows that real values of the first are lower than those of the second at any ratios L/mV and the difference between them decreases with L/mV. The efficiency E_{2mean} is higher than E_{3mean} in unreal regions when they are higher than unity and lower than zero.

The interrelations between E_{2mean} and E_{4mean} and also between E_{3mean} and E_{4mean} do not exist in a real range (Figs. 7 and 8); this impairs the importance of the model of mass transfer shown in Fig. 2d. It should be emphasized that the relationship between the efficiencies E_{2mean} and E_{4mean} (Fig. 7) does not depend on the parameter L/mV, i.e., it affects both of them equally.

It follows from the analysis made that the highest real values of the efficiency can be obtained using the model of mass transfer presented in Fig. 2c. For the region $L/mV > 1$ the efficiencies are arranged as $E_{3mean} > E_{1mean} > E_{2mean}$; within the range of $0.5 < L/mV < 1$ we have $E_{3mean} > E_{2mean} > E_{1mean}$. When $L/mV < 0.5$, a similar sequence is absent and only comparison of separate efficiencies is obtained: $E_{4mean} > E_{1mean}$ and $E_{3mean} > E_{2mean}$. It is of interest to note that if the efficiencies are higher than unity or lower than zero, then they are arranged in an inverse sequence: $E_{3mean} < E_{1mean} < E_{2mean}$ in the region $L/mV > 1$, $E_{3mean} < E_{2mean} < E_{1mean}$ within the range $0.5 < L/mV < 1$, and $E_{4mean} < E_{1mean}$ and $E_{3mean} < E_{2mean}$ when $L/mV < 0.5$.

Thus, all the suggested models of mass transfer can be used in the interpretation of test and industrial data, but the best comparison at various ratios of the flows of liquid and vapor with account for the coefficient of equilibrium is possible only for the versions presented in Fig. 2a-c.

742

NOTATION

E, efficiency of the plate; L, molar flow of the liquid; m, coefficient of equilibrium; V, molar flow of the vapor; x, y, concentrations of the volatile component in the liquid and the vapor, respectively. Super- and subscrips: fin, final portion; liq, liquid phase; in, initial portion; n, number of the considered plate; $n-1$, number of the previous plate streamwise with the vapor flow; mean, mean value; v, vapor phase; *, equilibrium state; 1-4, numbers of the considered versions of the change in the concentrations.

REFERENCES

1. E. V. Murphree, *Ind. Eng. Chem.*, 17, No. 7, 747-759 (1925).
2. H. Hausen, *Chem. Ing. Tech.*, 25, No. 10, 595-597 (1953).
3. K. Onda and O. Kobayashi, *Kogaku Kogaku*, 28, No. 10, 823-828 (1964).
4. K. Onda, E. Sada, K. Takahashi, and S. A. Mukhtar, *AIChE J.*, 17, No. 5, 1141-1152 (1971).
5. K. Onda, H. Takeuchi, and K. Takahashi, *J. Chem. Eng. Jpn.*, 5, No. 1, 13-19 (1972).
6. K. Onda, H. Takeuchi, K. Takahashi, and H. Matsuoka, *J. Chem. Eng. Jpn.*, 7, No. 5, 387-388 (1974).
7. A. G. Medina, N. Ashton, C. McDermott, *Chem. Eng. Sci.*, 34, No. 9, 1105-1112 (1979).